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**Contract:** N00014-18-P-2004 **Date:** 07/30/2018  
**Project Title:** Analysis and Prediction of Sea Ice Evolution using Koopman Mode  
Decomposition Techniques  
**Subject:** Monthly Progress Report  
**Period of Performance:** July 1, 2018 – July 30, 2018

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AIMdyn, Inc. respectfully submits Progress Report 4 for contract N00014-18-P-2004. Please direct any Technical questions on this report to the undersigned.

V/r

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# Business Status Report

- (1) Resource Status:  
Resourced to plan.
- (2) Contributions by AIMdyn Inc:  
See technical report.
- (3) Resource status VS Original schedule:  
On plan to original scope of work. Effort is fixed priced.

## PROGRAM FINANCIAL STATUS

Work Breakdown	Cumulative to Date	At Completion
CLIN#0001-CLIN#0004	\$74,748	\$112,125

Structure or Task Element	Planned Expend	Actual Expend	% Budget Compl	At Compl	Latest Revised Estimate	Remark Remark
CLIN#0004	\$18,687	\$18,687	66.7%	100 %	N/A	N/A

Subtotal: \$18,687

Management Reserve: N/A

Or Unallocated Resources: N/A

TOTAL: \$18,687

# Technical Status Report

## Abstract

Koopman Mode Decomposition of sea ice concentration measurements in the northern and southern hemispheres was used to predict future sea ice concentrations. Predicted values for time periods with existing data showed good agreement between the predicted and actual concentration values.

## ACCOMPLISHMENTS

### Summary

Existing measurements of sea ice concentration were used to produce predictions of sea ice concentration distributions in subsequent months using Koopman Mode Decomposition (KMD) of five year data windows. For those future months with existing data, the predicted concentration values were generally consistent with the existing data, with the accuracy of the predictions decreasing with time after the end of the time window used for KMD.

### Introduction

Koopman Mode Decomposition of a time-dependent signal into Koopman modes and eigenvalues permits reconstruction of the original signal, and also prediction of future values of the input signal beyond the time period used as an input to the KMD algorithm. In the case of sea ice measurements, it is of interest to predict the geographic distribution of observables of interest in future years.

Satellite measurements of sea ice concentration are available since 1979 and previous work on this program demonstrated the usefulness of examining five year windows of this data to capture multiyear dynamics. This approach permits the use of Koopman reconstruction techniques to compare actual data with the predicted sea ice concentration values for all but the most recent data.

### Methods

Reconstruction of the  $N_p$  sea ice concentration pixel values  $\mathbf{C}$  at discrete time step  $k$  is performed using the Koopman eigenvalues  $\lambda_j$  and the Koopman modes  $\mathbf{v}_j$  obtained from applying KMD to the concentration values over  $N$  time steps (months, in this case):

$$\mathbf{C}_k = \sum_{j=1}^N \lambda_j^{k-1} \mathbf{v}_j$$

Here, there are  $N$  Koopman eigenvalues and Koopman modes, where each Koopman eigenvalue is a single complex number and each Koopman mode has dimensions 1 by  $N_p$ .

For  $1 \leq k \leq N$ ,  $\mathbf{C}_k$  is termed a reconstruction of the  $k$ th time step in the original data  $\mathbf{C}$ , as the Koopman eigenvalues and modes came from a decomposition of the observations over

this time range and should simply reproduce the data used as input to the KMD. For  $k > N$ ,  $\mathbf{C}_k$  is a prediction of the future behavior of the sea ice concentration for the (future)  $k$ th time step, based on the system dynamics deduced from decomposition of earlier observations. This approach was applied to the concentration data for five year windows, where  $N = 60$  months, and prediction was performed for four years beyond that ( $k = 61$  to 108 months).

## Results and Discussion

Figures 1 to 8 show example northern hemisphere sea ice concentration prediction results compared with the actual sea ice concentration for the same month. The data shown are the predictions for January and July for the four years following the five year window 1979-1983 that was used as the input data for KMD. It is seen that the prediction results match the general extent and magnitude of the actual data as well as capturing many small scale features such as the shape of the concentration near the east coast of Greenland. The predictions are also seen to be less accurate in later months, particularly in specific details such as the concentration in the Sea of Okhotsk.

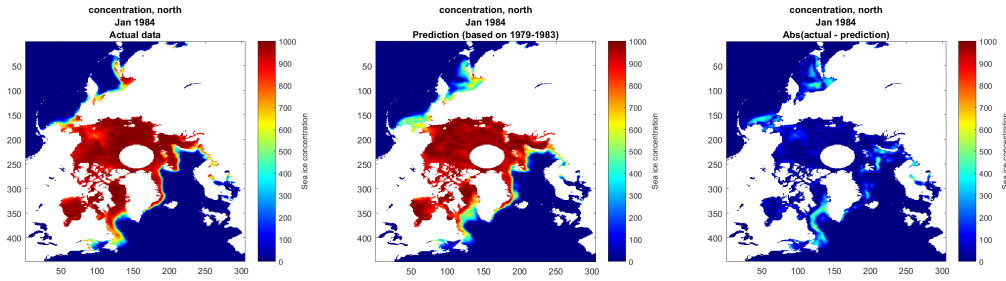


Figure 1: January 1984, 1 month after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

Figures 9 to 16 show example southern hemisphere sea ice concentration prediction results compared with the actual sea ice concentration for the same month. The data shown are the predictions for January and July for the four years following the five year window 1979-1983 that was used as the input data for KMD. Again, it is seen that the prediction results match the general extent and magnitude of the actual data as well as capturing many small scale features.

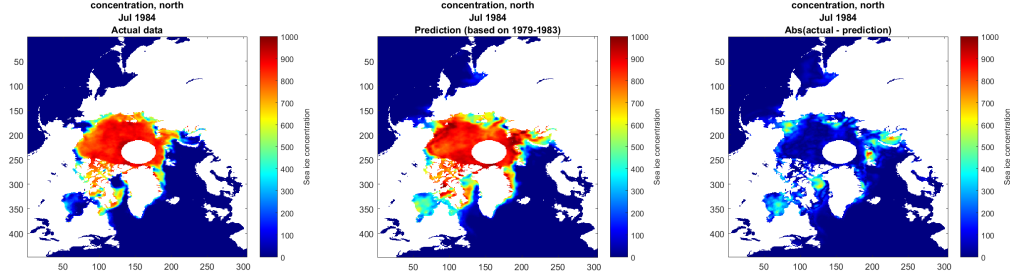


Figure 2: July 1984, 7 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

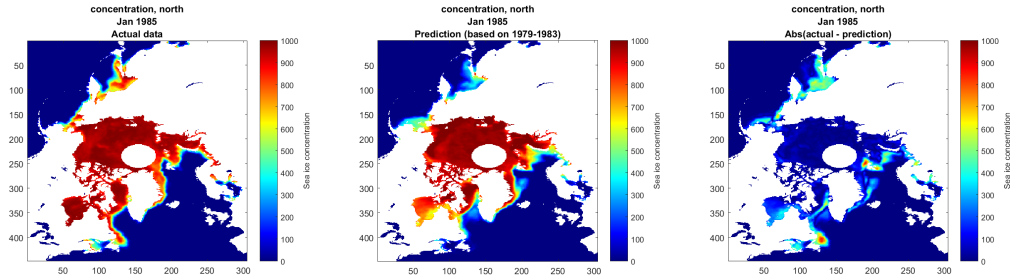


Figure 3: January 1985, 13 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

## Conclusions

Koopman Mode Decomposition was found to produce relatively accurate predictions over several years of future sea ice concentration values for five year data windows.

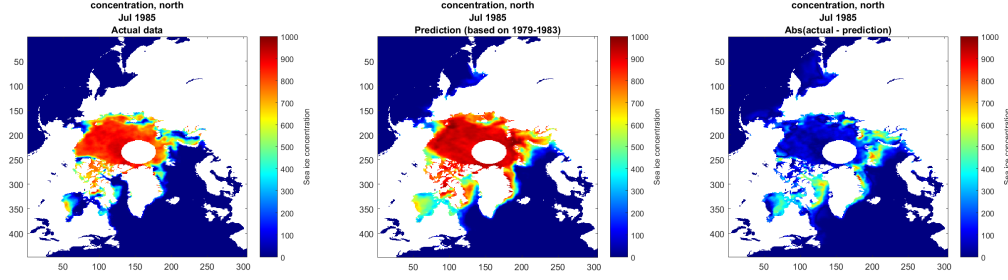


Figure 4: July 1985, 19 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

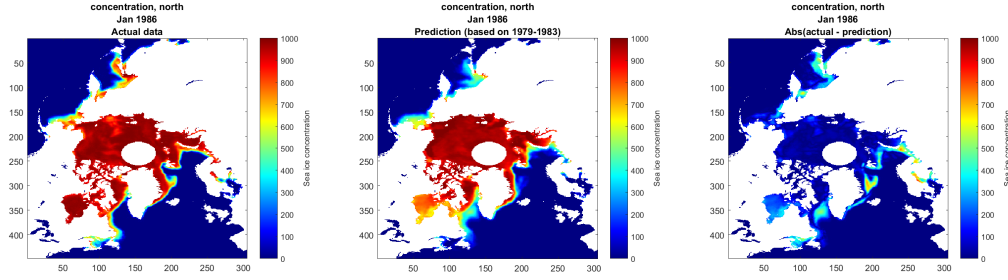


Figure 5: January 1986, 25 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

Future work is planned to consist of refinement of the reconstruction process and consideration of single-month year-to-year measurements.

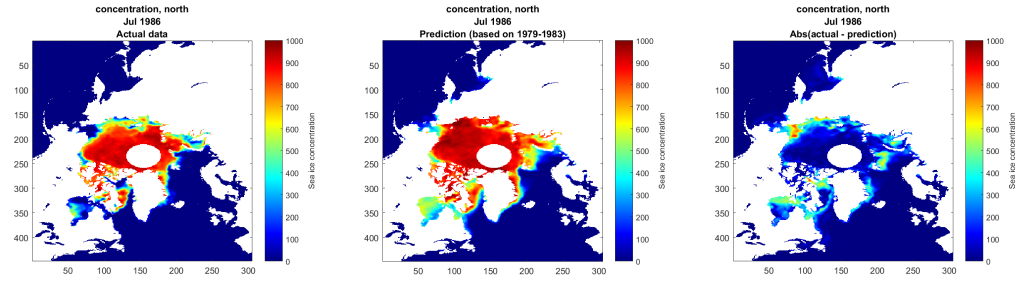


Figure 6: July 1986, 31 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

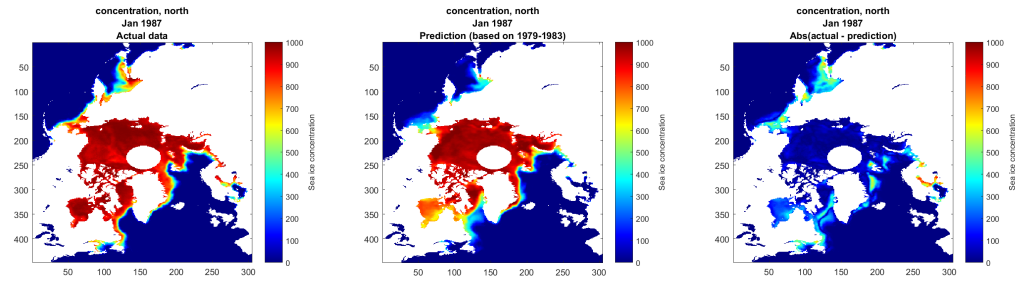


Figure 7: January 1987, 37 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

## Personnel Supported

Dr. Maria Fonoberova, Dr. Igor Mezic, Dr. James Hogg

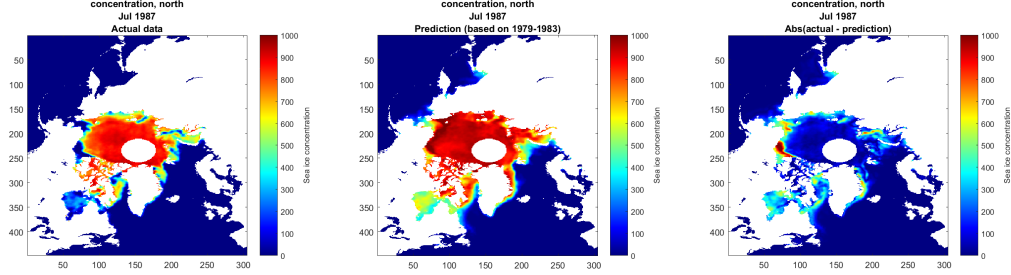


Figure 8: July 1987, 43 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

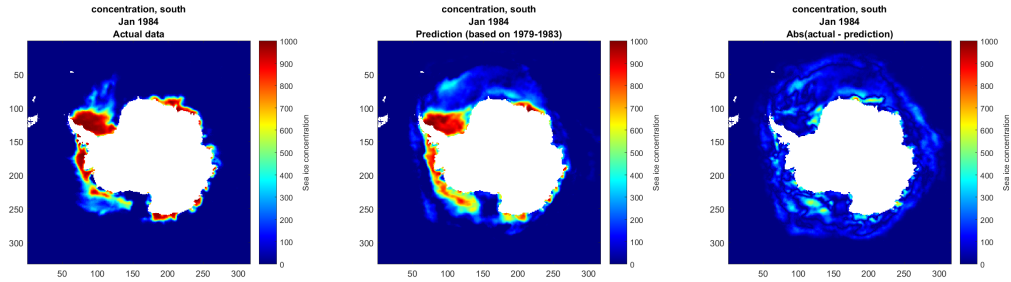


Figure 9: January 1984, 1 month after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration



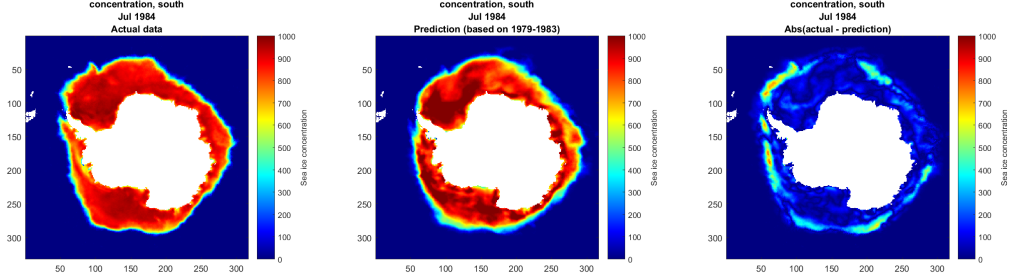


Figure 10: July 1984, 7 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

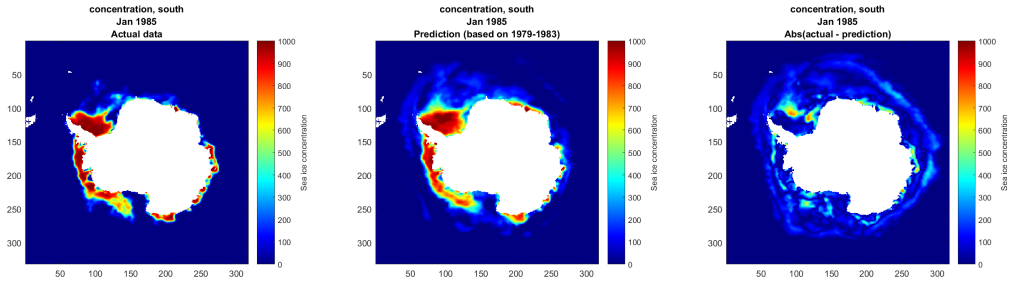


Figure 11: January 1985, 13 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

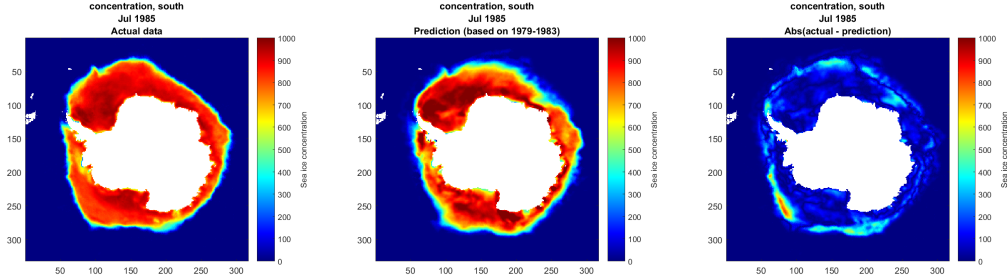


Figure 12: July 1985, 19 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

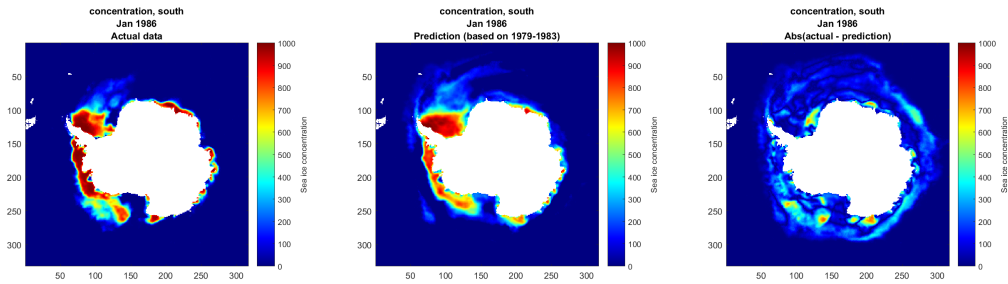


Figure 13: January 1986, 25 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

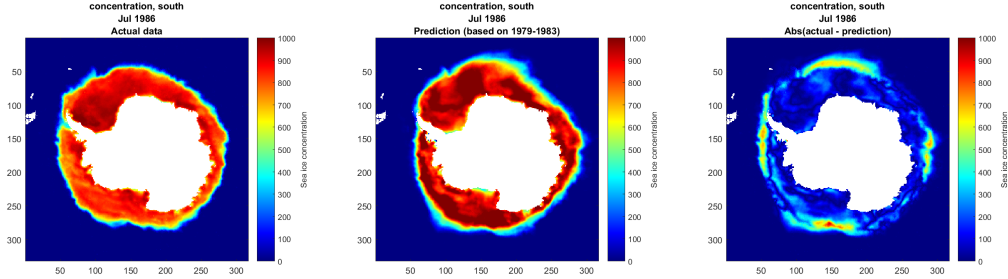


Figure 14: July 1986, 31 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

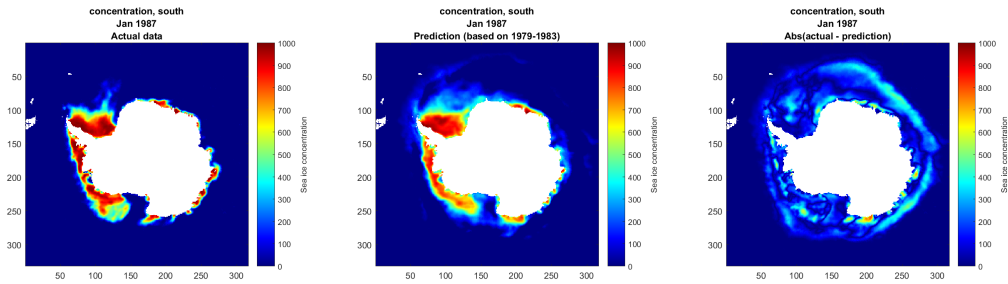


Figure 15: January 1987, 37 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration

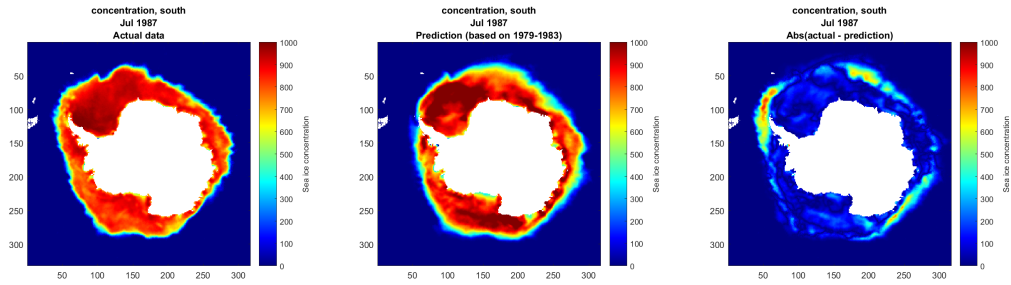


Figure 16: July 1987, 43 months after 1979-1983 input data. Left: actual concentration. Middle: predicted concentration. Right: Absolute difference between actual and predicted concentration